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Feedforward of Sampled-Data System for High-Precision Motion Control using Basis Functions with ZOH Differentiator

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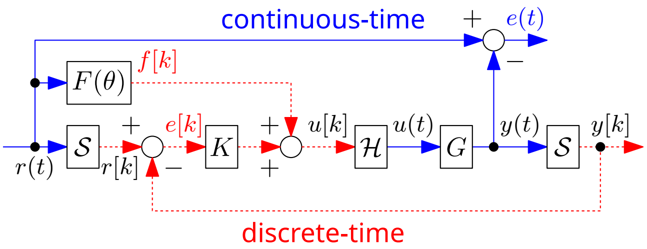
Background

Problem formulation



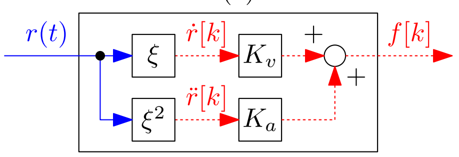
goal

design discrete-time feedforward $f[k]$ to improve tracking error in
- on-sample perfomance $e[k]$
- intersample performance $e(t)$

challenges

- consider ZOH characteristics in differentiator
- compensate for intersample oscillation [1]

Linearly parameterized feedforward

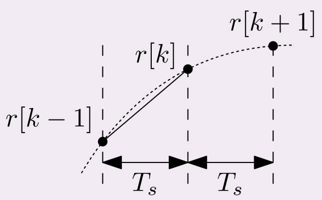


$$f(\theta)[k] = K_v\dot{r}[k] + K_a\ddot{r}[k]$$
$$= \underbrace{\begin{bmatrix}\xi r(t) & \xi^2 r(t)\end{bmatrix}}_{\Psi}\underbrace{\begin{bmatrix}K_v\\K_a\end{bmatrix}}_{\theta}$$

ξ: differentiator

- linearly parametrized with basis functions Ψ
- intuitive in physical meaning and easy for tuning θ

Conventional backward differentiator



$$\xi_{BD}^n = \left(\frac{1-z^{-1}}{T_s}\right)^n z^{\frac{n}{2}}\mathcal{S}$$

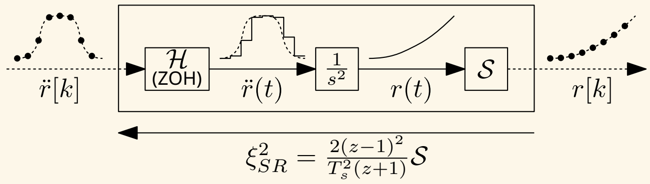
$\downarrow n=2$

$$\xi_{BD}^2 = \left(\frac{1-z^{-1}}{T_s}\right)^2 z\mathcal{S}$$

$\to$ not compatible with integrator and ZOH

Approach

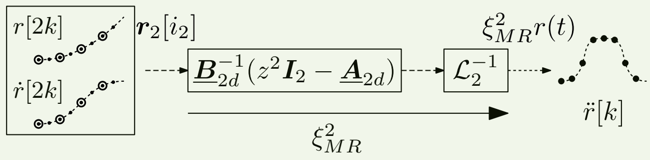
Differentiator in on-sample performance



$$\xi_{SR}^2 = \frac{2(z-1)^2}{T_s^2(z+1)}\mathcal{S}$$

- on-sample compatibility with integrator and ZOH
- no consideration for intersample performance

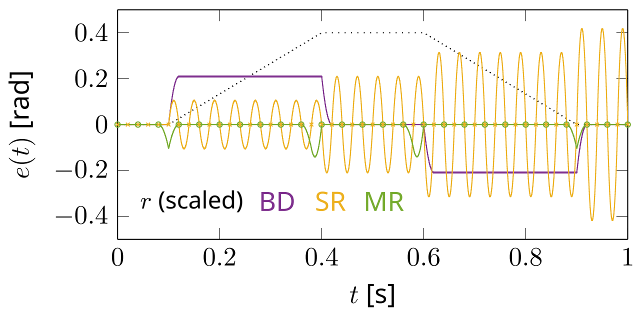
Differentiator in intersample performance



$$\frac{1}{s^2} = \left[\begin{array}{c|c}\boldsymbol{A}_{2c} & \boldsymbol{B}_{2c}\\ \hline \boldsymbol{C}_{2c} & 0\end{array}\right] = \left[\begin{array}{cc|c}0 & 1 & 0\\ 0 & 0 & 1\\ \hline 1 & 0 & 0\end{array}\right] \xrightarrow[(2T_s)]{\mathcal{L}_2\mathcal{S}(\frac{1}{s^2})\mathcal{H}} \left[\begin{array}{c|c}\underline{\boldsymbol{A}}_{2d} & \underline{\boldsymbol{B}}_{2d}\\ \hline \underline{\boldsymbol{C}}_{2d} & 0\end{array}\right] = \left[\begin{array}{c|cc}\boldsymbol{A}_{2d}^2 & \boldsymbol{A}_{2d}\boldsymbol{B}_{2d} & \boldsymbol{B}_{2d}\\ \hline \boldsymbol{C}_{2d} & 0 & 0\\ \boldsymbol{C}_{2d}\boldsymbol{A}_{2d} & \boldsymbol{C}_{2d}\boldsymbol{B}_{2d} & 0\end{array}\right]$$

- multirate ZOH differentiator for state compatibility [2]
- inversion of lifted system with integrator and ZOH

Results in 2nd order motion system



- perfect on-sample tracking with ZOH differentiator
- improvement of intersample performance with state tracking using multirate ZOH differentiator

Conclusion

	on-sample performance	continuous-time consideration	internal stability
backward differentiator (BD)	☹	☹	☺
single-rate ZOH differentiator (SR)	☺	☺	☹
multirate ZOH differentiator (MR)	😐 (every nT_s)	☺	☺

[1] T. Chen, & B. A. Francis, "Optimal Sampled-Data Control Systems." Springer London (1995)
[2] H. Fujimoto, Y. Hori, & A. Kawamura, "Perfect tracking control based on multirate feedforward control with generalized sampling periods." IEEE Transactions on Industrial Electronics (2001)